Analysis and Modeling of "Two-Gap" Coaxial Line Rectangular Waveguide Junctions

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Abstract —The analysis of "two-gap" coaxial line rectangular waveguide junctions is discussed. The cross-coupled junction, and the 'coax-gap' mount are specifically considered. The theoretical expressions obtained yield results in excellent agreement with published experimental results.

Equivalent circuits are presented for the two junctions applicable to the case where the TE_{10} mode is the only propagating waveguide mode.

I. Introduction

Cross-coupled coaxial line rectangular waveguide junctions (such as that shown in Fig. 1) have, over the years, been used in a wide variety of microwave devices. In the early days, they were used as a means of interconnecting coaxial line and rectangular waveguide, the junction being matched by the appropriate location of short circuits in one of the waveguide arms and one of the coaxial lines.

More recently, cross-coupled junctions have been used in IMPATT diode circuits, the active device being positioned in one of the coaxial lines, a resistive load (for stability purposes) terminating the other coaxial line, with a variable position short circuit in one of the waveguide arms providing a tuning adjustment. Many power combiners use such an arrangement as a basic module.

In the design of any of these devices, the microwave engineer needs to be able to calculate, with reasonable accuracy and usually over a significant frequency range, the input impedance at one port (of the four-port cross-coupled junction) for various load conditions in the other ports. To date, most design has been based on empirical knowledge, there having been relatively little research or analysis of such junctions reported in the literature. Only Lewin [1] and Eisenhart [2] have presented expressions, or outlined methods, by which impedance calculations could be made.

Lewin's analysis, which considers the junction viewed from one of the waveguide ports with the other ports perfectly matched, is based on the representation of the junction as a post with two delta function loads, located at the top and bottom of the post, the loads being of the same impedance as the characteristic impedance of the respective coaxial lines in the problem of interest. Because of the delta function load representation employed the impedance

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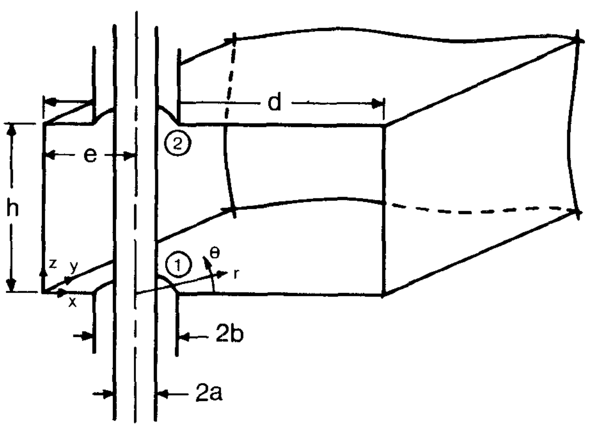

Fig. 1. A sectional view of the cross-coupled coaxial line rectangular waveguide junction.

result contains a divergent series. The result may be rendered finite [1], [3], but there remains the difficulty of taking into account the dimensions of the coaxial aperture of the practical problem. While reasonable results may be obtained in some cases without taking account of this aspect, as would appear to be so in the cases considered in [4, figs. 4 and 5] this would not be true for all cases. The input impedance at the coaxial port is known to be significantly affected by the dimensions of the coaxial aperture—as can be seen in [5, figs. 4 and 5].

Recognizing that the coaxial aperture dimensions have significant effect, Eisenhart and coauthors [5] proposed that the coaxial aperture be modeled as a finite gap (which they termed the equivalent gap), the width being a function of the dimensions of the problem. This equivalent gap was determined for C-band and X-band waveguide situations by the comparison of experimental results for the impedance of junctions of various dimensions, and theoretical results relating to the gap-excited post obtained from the analysis in [6]. The difficulty about this approach, in the general case, is the need to determine the equivalent gap which requires an extensive set of measurements.

Eisenhart obtained results for cross-coupled junctions [2] using this equivalent gap concept and his model of the two-gap waveguide mount which was deduced from his earlier analysis of the single-gap mount which is in turn based on representing the round post as an equivalent flat strip [6].

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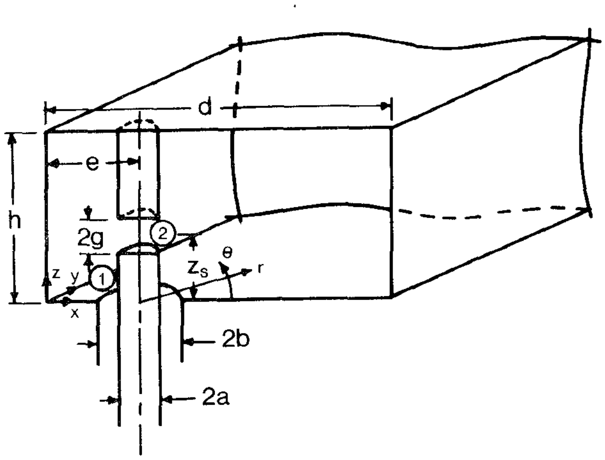

Fig. 2. A sectional view of the two-gap "coax-gap" mounting structure.

Thus, both the Lewin and Eisenhart formulations of the cross-coupled junction problem are indirect solutions of the practical problem.

There are of course other "two-gap" coaxial line rectangular waveguide junctions used in microwave circuits, and therefore of interest. One such junction is the "coax-gap" mounting structure shown in Fig. 2.

To date, only Eisenhart [2] has specifically considered this problem. His approach is identical to the one he adopted for the cross-coupled junction, that is, the use of his two-gap mount analysis and the representation of the coaxial source as an equivalent gap. Thus, as for the cross-coupled junction, we have an indirect solution of the practical problem.

This author's approach to the solution of the two "two-gap" coaxial junction problems discussed above is somewhat different. It involves the use of a two-port network approach and the formulations for two related problems, namely the single-coax entry junction and the single-gap mounting structure.

Because of the source models used and the method of solution, these formulations do not require the use of equivalence or correction factors. Consequently, the theoretical expressions may be used to obtain results for any particular (general) situation without having to first obtain, experimentally or otherwise, any additional information. The solutions for the "two-gap" problems, of course, inherit these characteristics.

In this paper it is shown how a two-port network approach may be adopted for the solution of the cross-coupled junction (Fig. 1) and the "coax-gap" mounting structure (Fig. 2), and the relevant equations are presented. Comparison of theoretical and experimental results show the theory to be very accurate.

II. Analysis of "Two-Gap" Junction Problems

"Two-gap" junctions, such as those shown in Figs. 1 and 2, may be analyzed as two-port network problems represented by the pair of equations

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2. \quad (1)$$

In a particular case, one must first define I and V appropriately, and having done so, the solution of the problem reduces to finding the Y_{11}, Y_{12}, Y_{21}, and Y_{22}. These may be obtained from the analysis of selected 'single-gap' problems. Of course, by reciprocity $Y_{12} = Y_{21}$.

Having found Y_{11}, Y_{21}, and Y_{22}, (1) can then be used to consider any case of interest. For example, the input admittance at port 1, Y_1, when port 2 is loaded with an impedance Z_{L2}, can be shown to be

$$Y_1 = Y_{11} - Z_{L_2}\frac{Y_{12}Y_{21}}{1+Z_{L_2}Y_{22}}. \quad (2)$$

In all cases, admittances and impedances are related to the aperture plane corresponding to the port in question.

In the problems considered in this paper, it is assumed that the waveguide and coaxial line(s) are perfectly conducting, and that only the TEM mode can propagate in the coaxial line(s) at the frequency of interest.

The analyses presented here are applicable to the case where the waveguide (assumed air-dielectric) is perfectly matched at both waveguide ports. The analyses can be used however, to deduce general equivalent circuits for the various problems considered. These equivalent circuits inter-relate the four ports of the problem (i.e., the two waveguide ports plus the two 'gap' ports), and are therefore useful for considering the junctions in a variety of loading situations. In this paper, equivalent circuits are presented for cases where the TE_{10} mode is the only propagating waveguide mode.

III. The Cross-Coupled Junction

Consider now the cross-coupled junction shown in Fig. 1, with ports 1 and 2 as shown.

The input admittance of the junction at either of the coaxial ports is given by an expression of the form [8]

$$Y = \frac{2\pi}{\ln(b/a)} \cdot \frac{\int_{\text{aperture}} H_\theta}{\int_{\text{aperture}} E_r}$$

where the integrals of the θ-component of the magnetic field (H_θ) and the radial component of the electric field (E_r) are over the coaxial aperture of the coaxial port at which the admittance is to be determined. Consistent with this admittance expression, I and V are taken to be (evaluated in respect to the appropriate aperture)

$$I = \frac{2\pi}{\ln(b/a)}\int_{\text{aperture}} H_\theta, \quad V = \int_{\text{aperture}} E_r.$$

Now if port 2 is short-circuited ($V_2 = 0$), the input admittance at port 1 (i.e., I_1/V_1) is simply Y_{11} and given by the result derived in [7], [8] for the single coax entry junction, namely (time dependence assumed $e^{j\omega t}$)

$$Y_{11} = \frac{2\pi j}{\eta_0 kh \ln^2(b/a)}\sum_{m=0}^{\infty}\epsilon_m\left\{D_m^{11} + \frac{\ln(b/a)}{q_m^2}\right\} \quad (3)$$

where $j = \sqrt{-1}$, $k = 2\pi/\lambda$, η_0 is the intrinsic impedance of free space, and

$$D_m^{11} = \begin{cases} -\frac{\pi}{2}\left[Y_0(\bar{q}_m ka)J_0(\bar{q}_m kb) - Y_0(\bar{q}_m kb)J_0(\bar{q}_m ka)\right] \\ \cdot\left[\frac{J_0(\bar{q}_m kb)}{J_0(\bar{q}_m ka)} + j\frac{J_0(\bar{q}_m kb)Y_0(\bar{q}_m ka) - J_0(\bar{q}_m ka)Y_0(\bar{q}_m kb)}{J_0(\bar{q}_m ka)S^*(\bar{q}_m ka, \bar{q}_m kd, e/d)}\right]/\bar{q}_m^2, & \frac{m\pi}{kh} < 1 \\ \left[K_0(q_m kb)I_0(q_m ka) - K_0(q_m ka)I_0(q_m kb)\right]\cdot\left[\frac{I_0(q_m kb)}{I_0(q_m ka)}\right. \\ \left. - \frac{I_0(q_m kb)K_0(q_m ka) - I_0(q_m ka)K_0(q_m kb)}{I_0(q_m ka)S(q_m ka, q_m kd, e/d)}\right]/q_m^2, & \frac{m\pi}{kh} > 1 \end{cases}$$

where

$$q_m = \sqrt{\left(\frac{m\pi}{kh}\right)^2 - 1}$$

$$\bar{q}_m = \sqrt{1 - \left(\frac{m\pi}{kh}\right)^2}, \quad \epsilon_m = \begin{cases} 1, & m = 0 \\ 2, & m \geq 1 \end{cases}$$

$$S^*(\bar{q}_m ka, \bar{q}_m kd, e/d) = H_0^{(2)}(\bar{q}_m ka) + J_0(\bar{q}_m ka) \cdot \left\{ \sum_{\substack{n=-\infty \\ \neq 0}}^{+\infty} H_0^{(2)}(2|n|\bar{q}_m kd) - \sum_{n=-\infty}^{+\infty} H_0^{(2)}(2|n + e/d|\bar{q}_m kd) \right\}$$

$$S(q_m ka, q_m kd, e/d) = K_0(q_m ka) + I_0(q_m ka) \cdot \left\{ \sum_{\substack{n=-\infty \\ \neq 0}}^{+\infty} K_0(2|n|q_m kd) - \sum_{n=-\infty}^{+\infty} K_0(2|n + e/d|q_m kd) \right\}$$

and J_0, Y_0, I_0, K_0, and $H_0^{(2)}$ are Bessel functions of the first and second kind, modified Bessel functions of the first and second kind, and Hankel functions of the second kind, respectively. (*Note*: The use of m in the above expressions has been chosen to be consistent with earlier publications of the author and others who have used similar methods for deriving such results. With the subsequent use of n, the nomenclature for the waveguide modes is thus TE_{nm} and TM_{nm}, where n relates to the field variation in the broad dimension of the waveguide, and m relates to that in the narrow dimension.)

Equation (3) may be rewritten as [8]

$$Y_{11} = -\frac{2\pi j}{\eta_0 kh \ln^2(b/a)} \cdot \left\{ kh \ln(b/a)\cot(kh) - D_0^{11} - 2\sum_{m=1}^{\infty} D_m^{11} \right\}. \quad (4)$$

The analysis of [8] may also be used to deduce the expression for Y_{21}, namely

$$Y_{21} = \frac{2\pi j}{\eta_0 kh \ln^2(b/a)} \sum_{m=0}^{\infty} \epsilon_m \left\{ (-1)^m \frac{\ln(b/a)}{q_m^2} + D_m^{21} \right\}$$

where

$$D_m^{21} = (-1)^m D_m^{11}.$$

Y_{21} may be rewritten as

$$Y_{21} = -\frac{2\pi j}{\eta_0 kh \ln^2(b/a)} \cdot \left\{ \ln(b/a)\cdot kh/\sin(kh) - D_0^{21} - 2\sum_{m=1}^{\infty} D_m^{21} \right\}. \quad (5)$$

Because the two coaxial lines have the same dimensions, it follows that $Y_{22} = Y_{11}$.

(*Note*: Details of the numerical evaluation of the various expressions are given in [8]–[10]. [9] and [10] may be obtained from the author.)

A. Comparison of Theoretical and Experimental Results for the Cross-Coupled Junction

As an illustration of the accuracy of the theoretical analysis, a comparison of theoretical and experimental results is given in Fig. 3 for the cross-coupled junction considered by Eisenhart [2]. In particular, results for the input impedance are shown for the situation: $a = 0.1525$ cm, $b = 0.35$ cm, $e/d = 0.5$, $h = 1.016$ cm, $d = 2.286$ cm for three different loads at port 2, namely 50 Ω, and short circuits 5 mm and 12 mm from the plane of the aperture of port 2. (The theoretical results were obtained directly from (2), (4), and (5) using the computer program listed in [10], while the experimental results are those of Eisenhart [2].) Clearly the agreement between theoretical and experimental results over the entire 7–16-GHz range is excellent. (It is interesting to compare Fig. 3, with [2, fig. 10] and [4, figs. 4 and 5]. The computational effort in the three different methods would be comparable.)

B. Equivalent Circuit

An equivalent circuit for the cross-coupled junction can be found by an analysis similar to that given in the Appendix. For the situation where the TE_{10} mode is the only propagating mode in the waveguide (a case of common interest,) the equivalent circuit is that shown in Fig. 4, where the susceptances B_a, B_b, and B_c are given by

$$B_a = B'_{11} - B'_{21}, \quad B_c = B'_{22} - B'_{21}$$

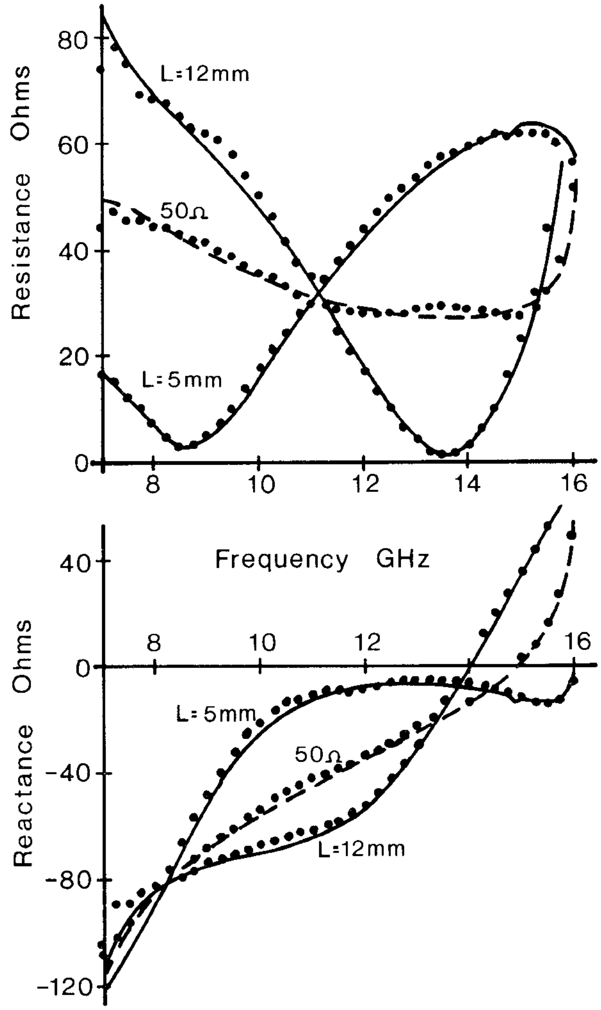

Fig. 3. A comparison of theoretical and experimental results for the impedance at one coax port of the cross-coupled junction when the other coax port is terminated in a 50-Ω load, and short circuits at 5 mm and 12 mm, respectively, and the waveguide ports are matched, for the case $a = 0.1525$ cm, $b = 0.35$ cm, $h = 1.016$ cm, $d = 2.286$ cm, and $e/d = 0.5$.——Theoretical results.····Experimental results.

and

$$B_b = B'_{21} - \frac{\pi^2}{\eta_0 kh \ln^2(b/a)} \frac{J_0(kb)}{J_0(ka)} \cdot (J_0(kb)Y_0(ka) - J_0(ka)Y_0(kb))$$

where

$$B'_{11} = B'_{22} = -\frac{2\pi}{\eta_0 kh \ln^2(b/a)} \cdot \left\{ \ln(b/a) kh \cot(kh) - 2 \sum_{m=1}^{\infty} D_m^{11} \right\}$$

and

$$B'_{21} = -\frac{2\pi}{\eta_0 kh \ln^2(b/a)} \left\{ \ln(b/a) kh / \sin(kh) - 2 \sum_{m=1}^{\infty} D_m^{21} \right\}$$

and R is given by

$$R = \frac{(2/\pi)\ln(b/a) J_0(ka) \sin(\pi e/d)}{J_0(ka)Y_0(kb) - J_0(kb)Y_0(ka)}.$$

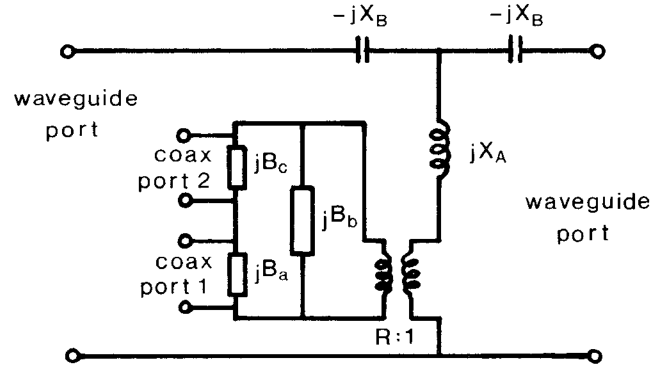

Fig. 4. The equivalent circuit of the cross-coupled junction for the case where the TE_{10} mode is the only propagating waveguide mode.

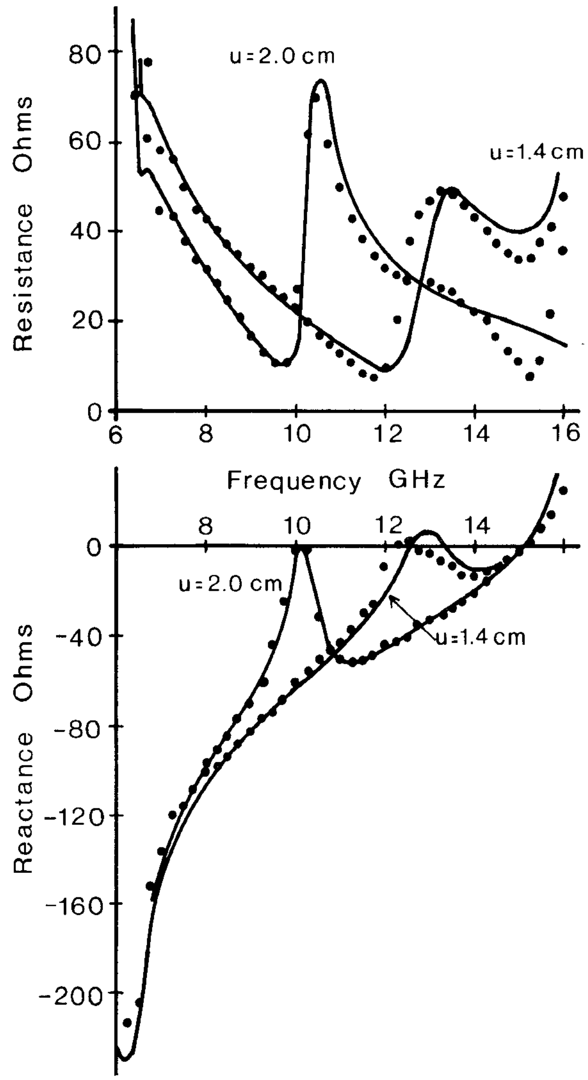

Fig. 5. A comparison of theoretical and experimental results for the input impedance at one coax port of the cross-coupled junction when the other coax port is terminated in 50 Ω, with one waveguide port matched and the other short circuited at distances 1.4 cm and 2.0 cm, respectively, for the case $a = 0.1525$ cm, $b = 0.350$ cm, $h = 1.016$ cm, $d = 2.286$ cm, and $e/d = 0.5$.——Theoretical results.····Experimental results.

C. Cross-Coupled Junction With A Short Circuit in One Waveguide Arm

The junction shown in Fig. 1 is often used in a situation where there is a short circuit in one of the waveguide arms, say a distance u from the center of the coaxial junction.

Results for this situation can be obtained from the equivalent circuit given in Fig. 4, provided the frequency is such that only the TE_{10} mode can propagate in the waveguide. (An alternative approach, applicable over a wider frequency range, is given in [10].)

The equivalent circuit has been used to calculate theoretical results for the input impedance at one of the coax ports of the cross-coupled junction for the following case: $a = 0.1525$ cm, $b = 0.350$ cm, $e/d = 0.5$, $h = 1.016$ cm, $d = 2.286$ cm, a 50-Ω load at the second coax port, for the two situations $u = 1.40$ cm and $u = 2$ cm. These theoretical results are shown in Fig. 5 together with experimental results obtained by Eisenhart [private communication]. Clearly the agreement between theory and experiment is most satisfactory.

IV. The 'Coax-Gap' Mounting Structure

The 'coax-gap' mounting structure shown in Fig. 2 can be analyzed in a similar way using the basic approach outlined in Section II.

Admittance calculations at the coaxial port have already been discussed. Thus, I_1 and V_1 (where ports 1 and 2 are defined in Fig. 2) are given by the expressions in Section III.

The input admittance at the gap port is given by an expression involving integrals of the tangential electric and magnetic fields in the gap aperture, namely [11]–[13]

$$Y = -\frac{2\pi a}{2g}\frac{\int_{\text{gap}} H_\theta}{\int_{\text{gap}} E_z}.$$

Consequently, I_2 and V_2 are defined by expressions of the form

$$V_2 = -\int_{\text{gap}} E_z$$

and

$$I_2 = \frac{2\pi a}{2g}\int_{\text{gap}} H_\theta$$

where the integrals are taken over the aperture of the gap at the surface of the post.

All that now remains is to determine Y_{11}, Y_{21}, and Y_{22}.

Y_{11} is the input admittance of a single coax entry junction and is given by (4), while Y_{22} (i.e., the input admittance of a single gap mounting structure) is given by [11]–[13]

$$Y_{22} = \frac{2\pi j}{\eta_0 kh}\left\{D_0^{22} + 2\sum_{m=1}^{\infty} D_m^{22}(\beta_m)^2\right\} \tag{6}$$

where

$$D_m^{22} = \begin{cases} -\left\{\dfrac{2j/\pi}{J_0(\bar{q}_m ka)S^*(\bar{q}_m ka, \bar{q}_m kd, e/d)} + \bar{q}_m ka\dfrac{J_1(\bar{q}_m ka)}{J_0(\bar{q}_m ka)}\right\}/\bar{q}_m^2, & \dfrac{m\pi}{kh} < 1 \\ \left\{\dfrac{1}{I_0(q_m ka)S(q_m ka, q_m kd, e/d)} - q_m ka\dfrac{I_1(q_m ka)}{I_0(q_m ka)}\right\}/q_m^2, & \dfrac{m\pi}{kh} > 1 \end{cases}$$

and

$$\beta_m = \cos\frac{m\pi z_s}{h}\frac{\sin\dfrac{m\pi g}{h}}{\dfrac{m\pi g}{h}}.$$

Y_{21} may be evaluated from the results in [8] and shown to be

$$Y_{21} = \frac{2\pi j}{\eta_0 kh\ln(b/a)}\left\{-\cos[kh(1 - z_s/h)]\cdot\frac{kh}{\sin kh}\cdot\frac{\sin kg}{kg} - D_0^{21} - 2\sum_{m=1}^{\infty} D_m^{21}\beta_m\right\} \tag{7}$$

where

$$D_m^{21} = \begin{cases} -\left[\dfrac{J_0(\bar{q}_m kb)}{J_0(\bar{q}_m ka)} + j\dfrac{J_0(\bar{q}_m kb)Y_0(\bar{q}_m ka) - J_0(\bar{q}_m ka)Y_0(\bar{q}_m kb)}{J_0(\bar{q}_m ka)S^*(\bar{q}_m ka, \bar{q}_m kd, e/d)}\right]/\bar{q}_m^2, & \dfrac{m\pi}{kh} < 1 \\ \left[\dfrac{I_0(q_m kb)}{I_0(q_m ka)} - \dfrac{I_0(q_m kb)K_0(q_m ka) - I_0(q_m ka)K_0(q_m kb)}{I_0(q_m ka)S(q_m ka, q_m kd, e/d)}\right]/q_m^2, & \dfrac{m\pi}{kh} > 1. \end{cases}$$

Details of the numerical evaluation of the various results have been given elsewhere [8]–[10], [12], [13].

This analysis may be used to deduce an equivalent circuit for the mount as outlined in the Appendix.

A. Comparison of Theoretical and Experimental Results

In order to demonstrate the application of the theory presented here, impedance calculations have been made (using (2), (4), (6), and (7)) for the mount considered by Eisenhart [2, fig. 9]. In particular, the calculations have been made for the case $a = 0.152$ cm, $b = 0.356$ cm, $h = 1.016$ cm, $d = 2.286$ cm, $e/d = 0.5$, $z_s = 0.696$ cm, and $g = 0.0889$ cm with the load in the gap being a varactor, taken to be represented by the model shown inset in Fig. 6 with $L_s = 0.45$ nH, $C_p = 0.2$ pF, and $R_s = 0.95$ Ω. The results of these calculations are shown in Fig. 6 for two cases corresponding to varactor bias voltages of 0 V ($C_j(0\text{ V}) = 1.9$ pF) and -30 V ($C_j(-30\text{ V}) = 0.45$ pF). Also shown in Fig. 6 are Eisenhart's experimental results.

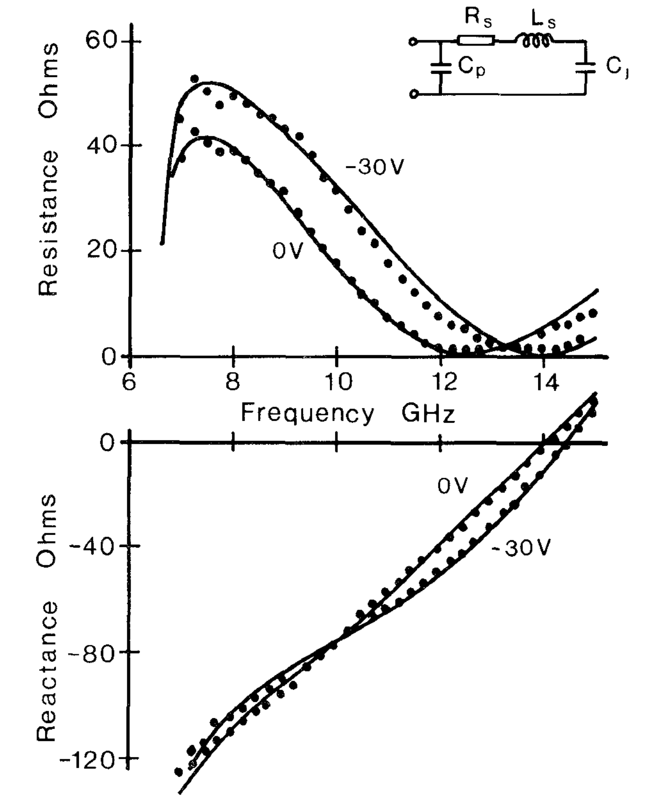

Fig. 6. Theoretical and experimental results for the input impedance at the coax port of the mount shown in Fig. 5 for the varactor gap load case described in the text.——Theoretical results.····Experimental results.

Clearly, the agreement between the experimental results and the theoretical results is very good.

V. Conclusion

The analysis of "two-gap" coaxial line rectangular waveguide junctions has been discussed.

The cross-coupled junction and the "coax-gap" mounting structure have been specifically considered. It has been shown that results computed directly from the theoretical expressions are in excellent agreement with experimental measurements.

Equivalent circuits have also been presented for the two junctions, applicable to the case where the TE_{10} is the only propagating waveguide mode.

Appendix

The admittance expressions presented in this paper can be used to derive equivalent circuits for the two junctions considered. Equivalent circuits are useful for considering situations in which the waveguide ports are mismatched, rather than the matched case to which the theory presented earlier in this paper relates.

In this Appendix, the equivalent circuit is deduced for the junction shown in Fig. 2 for the case where the frequency is such that the TE_{10} mode is the only propagating waveguide mode.

The equivalent circuit may be obtained by first representing (1) in the form of a π-network interconnecting the two ports, as shown in Fig. 7, and then rewriting Y_{11}, Y_{21},

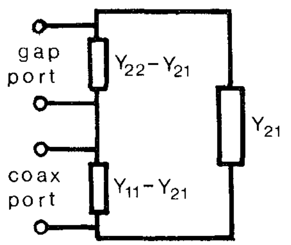

Fig. 7. π-equivalent circuit representation of (1).

and Y_{22} isolating the term relating to the TE_{10} mode.

Considering Y_{21}, for example, one can write

$$Y_{21} = \frac{4J_0(ka)}{\eta_0 khS^*(ka, kd, e/d)} \cdot \left[\frac{J_0(ka)Y_0(kb) - J_0(kb)Y_0(ka)}{(2/\pi)\ln(b/a)J_0(ka)}\right] \cdot \frac{1}{J_0(ka)} + Y'_{21} \quad (8)$$

where

$$Y'_{21} = \frac{2\pi j}{\eta_0 kh \ln(b/a)} \left\{ -\cos[kh(1 - z_s/h)] \cdot \frac{kh}{\sin kh} \frac{\sin kg}{kg} - 2\sum_{m=1}^{\infty} D_m^{21}\beta_m + \frac{J_0(kb)}{J_0(ka)} \right\}.$$

If the TE_{10} mode is the only propagating waveguide mode $S^*(ka, kd, e/d)$ may be rewritten as [8]

$$S^*(ka, kd, e/d) = \frac{8\sin^2(\pi e/d)J_0(ka)}{k_{10}d}\left(\frac{1}{2} + jx\right) \quad (9)$$

where

$$k_{10}d = \sqrt{(kd)^2 - \pi^2},$$

$$x = \frac{k_{10}d}{4\pi}\operatorname{cosec}^2\frac{\pi e}{d}\left\{2\sum_{n=2}^{\infty}\sin^2\frac{n\pi e}{d} \cdot \left[\frac{1}{\sqrt{n^2 - (kd/\pi)^2}} - \frac{1}{n}\right] - 2\sin^2\frac{\pi e}{d} + \ln\left[\frac{Ckd}{\pi}\sin\frac{\pi e}{d}\right] - \frac{\pi}{2}\frac{Y_0(ka)}{J_0(ka)}\right\}$$

and $C = 1.78107\cdots$.

From (8) and (9), we can rewrite Y_{21} as

$$Y_{21} = \frac{1}{Z_w(\frac{1}{2} + jx)} \cdot \frac{1}{R_1 R_2} + Y'_{21} \quad (10)$$

where

$$Z_w = \frac{2kh}{k_{10}d} \cdot \eta_0,$$

$$R_1 = \frac{(2/\pi)\ln(b/a)J_0(ka)\sin(\pi e/d)}{J_0(ka)Y_0(kb) - J_0(kb)Y_0(ka)}$$

and

$$R_2 = J_0(ka)\sin\frac{\pi e}{d}.$$

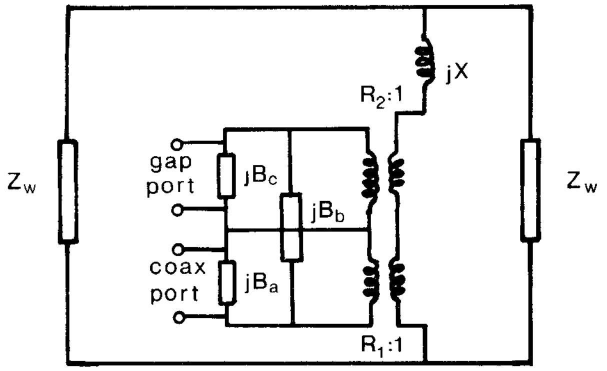

Fig. 8. Equivalent circuit with TE_{10} mode terms isolated.

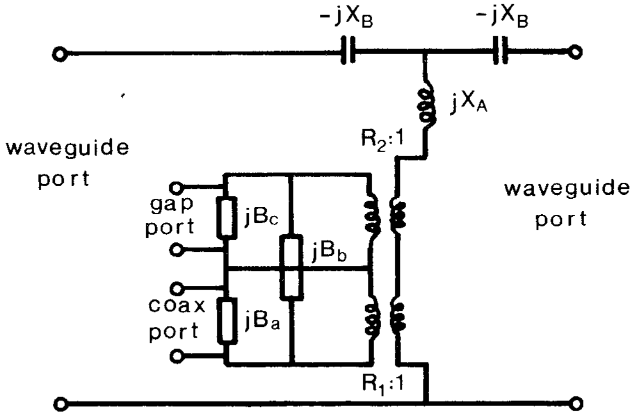

Fig. 9. Equivalent circuit for the mount shown in Fig. 5 for the case where the TE_{10} mode is the only propagating waveguide mode.

In a similar manner, one can show

$$Y_{11} = \frac{1}{Z_w\left(\frac{1}{2}+jx\right)} \cdot \frac{1}{R_1^2} + Y'_{11} \tag{11}$$

and

$$Y_{22} = \frac{1}{Z_w\left(\frac{1}{2}+jx\right)} \cdot \frac{1}{R_2^2} + Y'_{22} \tag{12}$$

where

$$Y'_{11} = -\frac{2\pi j}{\eta_0 kh \ln^2(b/a)} \cdot \left\{ \ln(b/a) \cdot kh \cot kh - 2 \sum_{m=1}^{\infty} D_m^{11} - \frac{\pi}{2} \cdot \frac{J_0(kb)}{J_0(ka)} \left[J_0(ka) Y_0(kb) - J_0(kb) Y_0(ka) \right] \right\}$$

and

$$Y'_{22} = \frac{2\pi j}{\eta_0 kh} \left\{ -ka \frac{J_1(ka)}{J_0(ka)} + 2 \sum_{m=1}^{\infty} D_m^{22} (\beta_m)^2 \right\}.$$

For the case being considered here, namely where the TE_{10} mode is the only propagating waveguide mode, Y'_{11}, Y'_{22}, and Y'_{21} are purely susceptive.

Note the common term $Z_w(\frac{1}{2}+jx)$ in (10), (11), and (12). The $\frac{1}{2}Z_w$ term arises because the two waveguide ports (assumed matched in the analysis) are being fed in parallel by the mount, while the $jxZ_w(=jX)$ term corresponds to the post reactance (cf., [14]).

Having recognized that the $\frac{1}{2}Z_w$ term is related to the TE_{10} (propagating) mode, we can now isolate the waveguide ports from the circuit of Fig. 7 (using (10)–(12)) and obtain the equivalent circuit shown in Fig. 8 where the susceptances B_a, B_b, and B_c are given by

$$jB_a = Y'_{11} - Y'_{21}$$
$$jB_b = Y'_{21}$$
$$jB_c = Y'_{22} - Y'_{21}.$$

Because the circuit was deduced from an analysis which had the two waveguide ports loaded identically (in fact, perfectly matched), the 'post-thickness' reactance (X_B) term is included in the X term. It is a simple matter to extract the X_B term, and obtain the equivalent circuit for the junction shown in Fig. 9, where

$$X_A = X + \frac{1}{2} X_B$$

and [14]

$$X_B = 2\pi \cdot k_{10} d \cdot (a/d)^2 \sin^2(\pi e/d) \cdot Z_w.$$

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Short Papers

Optimization of an Electrodynamic Basis for Determination of the Resonant Frequencies of Microwave Cavities Partially Filled with a Dielectric

JERZY KRUPKA

Abstract —**In this paper, a method of optimization of an electrodynamic basis is presented for determination of resonant frequencies of the microwave cavities containing dielectric samples. It is shown that the use of the suitable basis, consisting of several functions only, ensures a high accuracy of calculation of these frequencies.**

The presented method is useful for solving the boundary problem for the elliptic partial differential equation if the considered region has a regular boundary and is filled with inhomogeneous medium.

I. Theory

It is often necessary in practice to determine frequencies of the microwave resonant cavity in relation to the permittivity of the sample which fills this cavity. As it is known, this problem can be reduced to determination of the eigenvalues of the following boundary problem:

$$\begin{cases} L\phi = j\omega M\phi \\ \vec{n}\times\vec{E}=0 \text{ on } S \end{cases} \tag{1}$$

where

$$L=\begin{bmatrix} 0 & \vec{\nabla}x \\ \vec{\nabla}x & 0 \end{bmatrix},\quad M=\begin{bmatrix} \epsilon_0\hat{\epsilon}_r & 0 \\ 0 & -\mu_0 \end{bmatrix},\quad \phi=\begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix}$$

$\hat{\epsilon}_r$ is the relative complex permittivity inside the cavity, $\vec{E}, \vec{H}$ are the electric and magnetic fields inside the cavity, and S is the surface of the cavity.

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Eigenvalues ω of this problem can be accurately calculated if the sample fills completely two of the cavity dimensions. In other cases, approximation methods must be used. In the most accurate of them the electromagnetic field is expanded into a series

$$\phi=\sum_i \alpha_i\phi_i \tag{2}$$

where $\{\alpha_i\}$ is the set of coefficients to be determined and $\{\phi_i\}$ is the set of basis functions (the electrodynamic basis). If the electrodynamic basis is given, the well-known methods (e.g., the Rayleigh–Ritz or the Galerkin methods [1], [2]) are employed to calculate eigenvalues ω and eigenvectors $\{\alpha_i\}$. The main problem is how to find the best electrodynamic basis. Usually the basis contains functions which are solutions of the boundary problem (1) for the empty cavity. In this paper, the basis is formed by functions which are solutions of the boundary problem (1) for the cavity partially filled with a dielectric in a suitable manner. The dielectric fills completely two cavity dimensions. The cavity with such a filling is called the basis cavity.

The nature of such modification can be explained as follows. We want to achieve the best similarity of distributions of electromagnetic fields in the basis cavity (Fig. 1(b)) and in the cavity which fields we are looking for (Fig. 1(a)). In this particular case, shown in Fig. 1(b), we can achieve that by changing ϵ_b and (or) the radius r_b.

Similar modification of an electrodynamic basis was presented for the first time in [4] for the rectangular cavity with a rectangular dielectric sample where the authors assumed that $\epsilon_b=\mathrm{Re}(\hat{\epsilon}_r)$ = const. In this paper, generalizations are made by assuming any ϵ_b value and by optimization of the choice of particular basis